Module 3

- **Union**

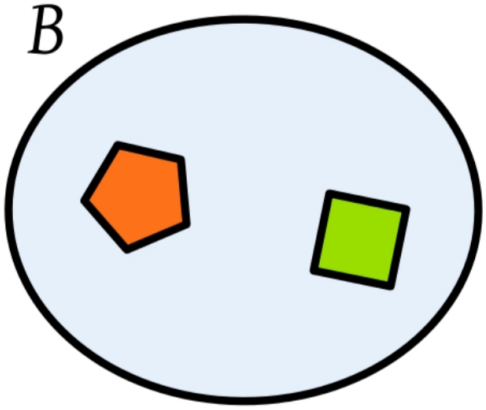
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- **Intersect^{n}**

$A =$

$B =$

$A \cap B$



- **Disjoint (Mutually Exclusive) Event**

$$P(A \cap B) = \varphi$$

- **Independent Event**

$$P(A \cap B) = P(A) \,.\, P(B)$$

  - **Ex. of Dependent & Independent Event**

    "Two cards are drawn from a pack of cards where the 1st card IS placed back in the pack before drawing the 2nd card" *is an independent event*. "Two cards are drawn from a pack of cards where the 1st card is NOT placed back in the pack before drawing the 2nd card" *is a dependent event*.

- **N.B.**

The **difference between mutually exclusive & independent event** is that, a mutually exclusive event can simply be defined as a situat^{n} when 2 events can't occur at the same time, whereas, an independent event occurs when 1 event remains unaffected by the occurrence of the other event.

Ex: When a coin is tossed, then there're 2 events that can occur. Either it will be a head or a tail. Hence, both the events here are mutually exclusive. But if we take 2 separate coins & flip them together, then the occurrence of Head or Tail on both the coins are independent of each other.

- **Complement Event**

$$P(A^c) = 1 - P(A)$$

These events occur when there're only 2 outcomes, e.g. passing or failing an exam.The complement means the exact opposite of an event.

- **Properties:**

$$\text{If } A \subseteq B, \text{ then } P(A) \leq P(B)$$

$$P(A \cup B) = P(A) + P(B - A) = P(A \cup (A^c \cap B))$$

- **N.B.**

$$P(A^c \cap A) = \varphi$$
$$P(A^c \cap B) = B$$

All complement events are mutually exclusive events, but the reverse isn't always true.

- **Difference**

$$P(A - B) = P(A) - P(A \cap B)$$

- **Corollary**

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

- **Condit^{n}al probability**

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

- **Multiplicat^n Rule**

$$P(E \cap F) = P(F \mid E) \,.\, P(E) = P(E \mid F) \,.\, P(F)$$

- **Bayer's Formula**

$$P(E) = [P(E \mid F) \,.\, P(F)] + [P(E \mid F^c) \,.\, P(F^c)]$$

- **Odds of an Event**

$$\frac{P(A)}{P(A^c)} = \frac{P(A)}{1 - P(A)}$$

- **New odds after the evidence is introduced**

$$\frac{P(H \mid E)}{P(H^c \mid E)} = \frac{P(E \mid H) \,.\, P(H)}{P(E \mid H^c) \,.\, P(H^c)}$$

- **Probability Multiplicat^{n} Rule**

$$P(E) = \sum_{i=1}^{n} P(E \mid F_i) \,.\, P(F_i)$$

Also called **Law of Total Probability**.

- **Baye's Th^{m} (Formula)**

$$P(F_j \mid E) = \frac{P(E \cap F_j)}{P(E)} = \frac{P(E \mid F_j) \,.\, P(F_j)}{\sum_{i=1}^{n} P(E \mid F_i) \,.\, P(F_i)}$$

- **Independent Events**

$$P(E \mid F) = P(E)$$
$$P(F \mid E) = P(F)$$

$$P(E \cap F \cap G) = P(E) \,.\, P(F) \,.\, P(G)$$

  - **N.B.**

$$P\left(\bigcap_{n=1}^{k} E_n\right) = \prod_{n=1}^{k} P(E_n)$$

- **Property**

$$P(A \cup B) = 1 - [P(A^c) \,.\, P(B^c)]$$

  - **N.B.**

    If 2 events are independent, then their complements are also independent.

- **Random Variables**

$$P\left(\bigcup_{i=(N/W)}^{n} (X = i)\right) = \sum_{i=(N/W)}^{n} P(X = i)$$

- **CDF**

$$F(x) = P\{X \leq x\}$$

- **PMF (Probability Distribut^n) (PMF of DRV)**

$$P(a) = P\{X = a\}$$

  - **Property**

$$\sum_{i=0}^{\infty} P(x_i) = 1$$

- **CDF in terms of PMF**

$$F(a) = \sum_{0 \leq x \leq a} P(x)$$

- **Expected value (Mean) of a Random Variable**

$$E[X] = \sum_{P(x_i) > 0} x_i P(x_i)$$

Also called **1st moment of X**.

- **Property**

$$(E[X])^2 < E[X^2]$$

- **Proposit^n**

$$E[g(X)] = \sum_{i=1}^{n} g(x_i) P(x_i)$$

$$E[X^n] = \sum_{i=1}^{n} x_i^n P(x_i)$$

Also called **nth moment of X**.

- **Corollary**

$$E[aX + b] = aE[X] + b$$

- **N.B.**

$$E[X] = \mu$$

- **Variance of a Random Variable**

$$Var(X) = E[(X - \mu)^2]$$

$$Var(X) = E[X^2] - (E[X])^2$$

- **N.B.**

$$Var(aX + b) = a^2 Var(X)$$

- Std. Deviat^n

$$SD(X) = \sigma = \sqrt{Var(X)}$$